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Critical Field Length Calculations for Preliminary Design

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Two methods are presented for determining the critical field length for multiple-engine jet aircraft during the preliminary design process. The first method includes the effects of thrust deflection, thrust variation with velocity, and head or tail winds. The second approximation is based upon zero wind and constant thrust values, and has been programmed on a personal calculator. While the principal application of this method has been to military aircraft, the method can be applied to civil aircraft performance when suitably modified.

Nomenclature

A = dimensional coefficient of Eq. (5), 1/ft
B = dimensional coefficient of Eq. (5), 1/s
C = dimensional coefficient of Eq. (5), ft/s^2
C_D = drag coefficient
C_L = lift coefficient
D = drag, lb
d = differential operator
e = base of natural logarithms
E = intermediate function defined for Eq. (15), nondimensional
F = intermediate function defined for Eq. (22), nondimensional
G = scale length defined for Eq. (16), ft
g = acceleration of gravity, ft/s^2
k = stall speed multiplier used to define takeoff speed
L = lift, lb
R = intermediate function defined for Eq. (7), 1/s^2
S = wing reference area, ft^2
T = thrust, lb
T_0, T_1, T_2 = thrust equation coefficients
t = time, s
V = velocity, ft/s
$\dot{V}$ = acceleration, ft/s^2
$\ddot{V}$ = time derivative of acceleration, ft/s^3
$\bar{V}$ = velocity for $\dot{V}=0$, ft/s
W = weight, lb
w = wind velocity, ft/s
X = distance, ft
δ = thrust deflection angle, deg
Δt = time increment required for making and implementing the continue/stop decision
ΔX_{OBS} = distance from liftoff to obstacle clearance
μ = coefficient of friction, nondimensional
ρ = density, (lb s^2)/ft^4

Subscripts

a = initial velocity, time or distance
b = final velocity, time or distance
OEI = one engine inoperative
TO = takeoff condition

Introduction

THE determination of the critical field length and critical engine failure speed for multiengined military jet aircraft can be a difficult and complex task. However, in the preliminary design phase quick solutions based upon reasonable approximations are highly desirable. A scan of the literature reveals a paucity of such solutions. Nicolai[1] defines the problem and recommends graphical solutions. Torenbeek[2,3] provides a thorough discussion of the problem and generates a closed-form expression for balanced field length based upon the use of average accelerations in each phase of the takeoff. Krenkel and Salzman[4] discuss the calculation of takeoff distances for conventional and vectored thrust STOL aircraft without addressing the problems of critical field length.

The determination of the distance required to accelerate (or decelerate) to a given speed is the cornerstone of the type of analysis. Many analyses have been carried out using different models for the variation of thrust and velocity. Diehl[5] used a linear variation in 1932. Hartman[6] assumed that the acceleration varied inversely with V^2. Krenkel and Salzman[4] used a constant thrust approximation after showing that a quadratic variation in thrust produced only very small changes in takeoff distance for their type of vehicle.

This paper shows that the task of determining critical field lengths and critical engine failure speeds can be reduced to the solution of a simple transcendental equation through the use of a minimum number of approximations. Two levels of approximations are discussed. The first provides for thrust deflection, thrust variation with relative velocity, and the effects of head or tail winds. The second approximation uses a fixed average thrust for each segment of the takeoff process and disregards wind effects. The latter technique has been programmed on a personal calculator, and provides results in close agreement with the first method.

The Problem

The critical field length for a given takeoff weight of an aircraft is defined as that distance required to make either a one-engine-inoperative takeoff or a braked stop when an engine fails at the critical engine failure speed.

Mil Spec MIL-M-007700B (USAF) states in part, "...The critical field length shall be based on the following rules.

"a. At engine failure speed the aircraft continues to accelerate for 3 seconds with remaining engines at maximum thrust and zero thrust on the inoperative engine.

"b. At the end of the 3-second acceleration time, thrust on all engines is instantaneously reduced to idle, brakes applied and deceleration devices deployed."

Note that takeoff speed is usually defined as kV_{stall}, where k is usually taken to be 1.1 although other values can be used. Figure 1 defines the problem. Curve 1 describes the velocity-distance relationship of a normal takeoff run with all engines operating. At the critical engine failure speed V_1, the loss of an engine reduces the acceleration capability of the aircraft; the resulting time history is shown as curve 2. The aircraft requires a specified time interval Δt to reach the velocity V_2. By definition, the distances required to stop or to takeoff with

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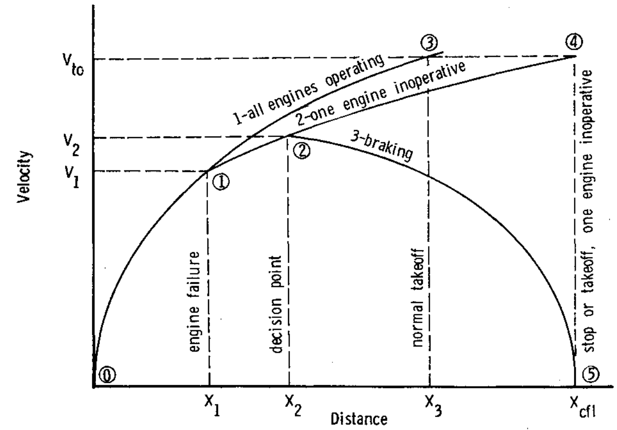

Fig. 1 Velocity-distance relationships for critical field length.

one engine inoperative from V_2 are identical. The acceleration time history is described by curve 2, the deceleration time history by curve 3. The problem, then, is to determine the total distance required and the critical engine failure speed V_1. Note that the one-engine-inoperative takeoff distance usually includes the distance required to clear an obstacle of a specified height at a specified multiple of the aircraft stall speed.

Comprehensive Approach

The instantaneous acceleration acting on an aircraft during the takeoff process is:

$$\frac{dV}{dt}=\frac{T\cos\delta-[D+\mu(W-L-T\sin\delta)]}{W/g} \tag{1}$$

The aircraft thrust variation with airspeed in the takeoff speed range can be adequately represented by a quadratic equation. Let the wind component acting along the flight path be w (positive for head winds) and the aircraft inertial velocity be V. The thrust representation is then,

$$T=T_0+T_1(V+w)+T_2(V+w)^2 \tag{2}$$

where the coefficients T_0, T_1, and T_2 are determined by a curve fit of engine thrust data.

The aircraft drag force is given by

$$D=\tfrac{1}{2}\rho SC_D(V+w)^2 \tag{3}$$

and the lift force by

$$L=\tfrac{1}{2}\rho SC_L(V+w)^2 \tag{4}$$

Substituting Eqs. (2-4) into Eq. (1), expanding, and collecting like powers of V, we find that

$$\frac{dV}{dt}=\dot{V}=AV^2+BV+C \tag{5}$$

where

$$A=\left[(\cos\delta+\mu\sin\delta)T_2-(C_D-\mu C_L)\frac{\rho S}{2}\right]\frac{g}{W}$$

$$B=\left[(\cos\delta+\mu\sin\delta)(T_1+2T_2w)-(C_D-\mu C_L)\rho Sw\right]\frac{g}{W}$$

$$C=\left[(\cos\delta+\mu\sin\delta)(T_0+T_1w+T_2w^2)-\mu W-(C_D-\mu C_L)\frac{\rho Sw^2}{2}\right]\frac{g}{W}$$

For later use, note that

$$\ddot{V}=2AV+B \tag{6}$$

If we assume that the lift and drag coefficients, the weight, the friction coefficient, and the thrust deflection angle remain constant, the coefficients A,B, and C of Eq. (5) are themselves constants and Eq. (5) is directly integrable. Rearranging Eq. (5) and integrating, we have:

$$\int_{t_a}^{t_b}dt=\int_{V_a}^{V_b}\frac{dV}{AV^2+BV+C}$$

$$=\frac{1}{\sqrt{R}}\ln\left\{\left|\frac{\ddot{V}_b-\sqrt{R}}{\dot{V}_b+\sqrt{R}}\,\frac{\ddot{V}_a+\sqrt{R}}{\dot{V}_a-\sqrt{R}}\right|\right\}\qquad R\equiv B^2-4AC>0 \tag{7}$$

$$=\frac{2}{\sqrt{-R}}\left[\arctan\left(\frac{\ddot{V}_b}{\sqrt{-R}}\right)-\arctan\left(\frac{\ddot{V}_a}{\sqrt{-R}}\right)\right]\qquad R<0 \tag{8}$$

In order to achieve takeoff an aircraft must be able to accelerate from a standing start to the takeoff velocity. As the aircraft speed increases, the acceleration capability decreases due to the increase in drag and changes in thrust, and goes to zero at some speed usually far above the takeoff speed. For this situation R will be greater than zero indicating that dV/dt has a real root. However, for the deceleration case where the engines are at idle, the acceleration capability is usually entirely negative. Thus, Eq. (5) may have no real root, in which case R will be negative. Hence, both Eqs. (7) and (8) are needed in any evaluation.

The order to relate distance and velocity, the variable dt in Eq. (5) is replaced by $dX(dt/dX)=(1/V)dX$. Using this in Eq. (5) and rearranging, we have

$$dX=\frac{VdV}{AV^2+BV+C} \tag{9}$$

As before, Eq. (9) can be directly integrated if A,B, and C are constants. Hence

$$X_b-X_a=\frac{1}{2A}\ln\left[\left|\frac{\dot{V}_b}{\dot{V}_a}\right|\right]-\frac{B}{2A}(t_b-t_a) \tag{10}$$

where t_a-t_b are evaluated using Eq. (7) or (8), as appropriate.

Referring back to Fig. 1, the distance required to accelerate from V_2 to V_{TO} along the OEI curve must be the same as the distance required to accelerate from V_2 to $V=0$ along the braked curve. Let primary subscripts on $\dot{V}$, $\ddot{V}$, and R denote the point and the secondary subscripts denote the curve at and on which they are to be evaluated. For example, $\dot{V}_{23}$ denotes Eq. (5) evaluated at point 2 on curve 3. The statement of equality of distance to takeoff to distance to stop is then given by

$$\frac{1}{2A_3}\ln\left[\left|\frac{\dot{V}_{53}}{\dot{V}_{23}}\right|\right]-\frac{B_3}{2A_3}(t_5-t_2)=\frac{1}{2A_2}\ln\left[\left|\frac{\dot{V}_{43}}{\dot{V}_{22}}\right|\right]-\frac{B_2}{2A_2}(t_4-t_2)+\Delta X_{\text{OBS}} \tag{11}$$

where

$$t_4-t_2=\frac{1}{\sqrt{R_2}}\ln\left[\left|\frac{\ddot{V}_{42}-\sqrt{R_2}}{\dot{V}_{42}+\sqrt{R_2}}\,\frac{\dot{V}_{22}+\sqrt{R_2}}{\dot{V}_{22}-\sqrt{R_2}}\right|\right]\qquad R_2>0 \tag{12}$$

$$t_5-t_2=\frac{1}{\sqrt{R_3}}\ln\left[\left|\frac{\ddot{V}_{53}-\sqrt{R_3}}{\dot{V}_{53}+\sqrt{R_3}}\,\frac{\dot{V}_{23}+\sqrt{R_3}}{\dot{V}_{23}-\sqrt{R_2}}\right|\right]\qquad R_3>0 \tag{13}$$

or

$$t_5 - t_2 = \frac{1}{\sqrt{-R_3}}\left[\arctan\left(\frac{\ddot{V}_{53}}{\sqrt{-R_3}}\right) - \arctan\left(\frac{\ddot{V}_{23}}{\sqrt{-R_3}}\right)\right] \qquad R_3 < 0 \tag{14}$$

Note that the ground distance required from liftoff to obstacle passage is added to the right side of Eq. (11). This increment is usually obtained by integrating the equations of motion, and for a given aircraft and a given set of atmospheric conditions will be a constant. Also note that while the lift, drag, and friction coefficients are constants, different values may be used to generate curves 1,2, and 3.

Careful examination of this set of equations reveals that V_2 is the only unknown. (V_{TO} is taken to be kV_{stall}.) While seemingly complex this set of equations can be solved quickly and easily for V_2 by an iteration process.

Using Eq. (7) and evaluating between V_1 and V_2 for $t_b - t_a = \Delta t$, we find that

$$V_1 = \frac{B_2(1+E) - \sqrt{R_2}(1-E)}{2A_2(1-E)} \tag{15}$$

where

$$E \equiv \pm \frac{\ddot{V}_{22} + \sqrt{R_2}}{\ddot{V}_{22} - \sqrt{R_2}} e^{\sqrt{R_2}\Delta t}$$

The problem is now completely determined. The value of V_2 is determined by Eq. (11) and its ancillaries, V_1 is determined by Eq. (15), and the variation of velocity with distance by using Eq. (7) or (8), as appropriate. The variation of distance with velocity is given by Eq. (10). Curve 1 velocities vary from $V=0$ through V_{TO}, curve 2 velocities vary from V_1 through V_{TO}, and curve 3 velocities vary from V_2 to $V=0$.

Simplified Approach

If the effects of head wind and thrust deflection are disregarded, and the thrust is assumed constant at some representative value, a simplified set of equations can be developed. This set can easily be programmed for use on a programmable personal calculator such as HP 67 or TI 59.

Let the constant value of thrust be T. The values of the coefficients of Eq. (5) then become:

$$A = \left[-(C_D - \mu C_L)\frac{\rho S}{2}\right]\frac{g}{W} \equiv -\frac{1}{2G}$$

$$B = 0$$

$$C = \left[T(\cos\delta + \mu\sin\delta) - \mu W\right]\frac{g}{W}$$

and Eq. (5) itself becomes:

$$\frac{\mathrm{d}V}{\mathrm{d}t} = C - \frac{1}{2G}V^2 \tag{16}$$

Equation (16) integrates to

$$t_b - t_a = \frac{G}{\bar{V}}\ln\left[\left|\frac{\bar{V}+V_b}{\bar{V}-V_b}\,\frac{\bar{V}-V_a}{\bar{V}+V_a}\right|\right] \tag{17}$$

where

$\bar{V}$ $=\sqrt{2G}=$ velocity for $\mathrm{d}V/\mathrm{d}t = 0$
V_b $=$ final velocity
V_a $=$ initial velocity

As before, by replacing $\mathrm{d}t$ by $\mathrm{d}X/V$, Eq. (16) becomes

$$\mathrm{d}X = \frac{V\mathrm{d}V}{C - \frac{1}{2G}V^2} \tag{18}$$

which integrates to

$$X_b - X_a = -G\ln\left|\frac{\bar{V}^2 - V_b^2}{\bar{V}^2 - V_a^2}\right| \tag{19}$$

By equating the distance from V_2 to takeoff along curve 2 to the distance from V_2 to a complete stop along curve 3 we have

$$-G_3\ln\left[\left|\frac{\bar{V}_3^2}{\bar{V}_3^2 - V_2^2}\right|\right] = -G_2\ln\left[\left|\frac{\bar{V}^2 - V_4^2}{\bar{V}_2^2 - V_2^2}\right|\right] + \Delta X_{\mathrm{OBS}} \tag{20}$$

The only unknown in Eq. (20) is now V_2. Equation (20) is less complex than Eq. (11), and can also be easily solved by an iteration technique. From Eq. (17) we have

$$\Delta t = \frac{G_2}{\bar{V}_2}\ln\left[\left|\frac{\bar{V}_2+V_2}{\bar{V}_2-V_2}\,\frac{\bar{V}_2-V_1}{\bar{V}_2+V_1}\right|\right] \tag{21}$$

which can be solved for V_1 since V_2 is known. Let

$$F = \pm\left(\frac{\bar{V}_2 - V_2}{\bar{V}_2 + V_2}\right)\exp\left(\frac{\bar{V}_2}{G_2}\Delta t\right)$$

then

$$V_1 = \left(\frac{1-F}{1+F}\right)\bar{V}_2 \tag{22}$$

which corresponds to Eq. (15). The problem is now completely solved. The value of V_2 is given by the solution of Eq. (20) and V_1 by Eq. (22). The distance-velocity relationship is given by Eq. (19), and the time-velocity relationship by Eq. (17). As before curve 1 extends from $V=0$ to V_{TO}, curve 2 from V_1 to V_{TO} and curve 3 from V_2 to $V=0$.

Examples

The characteristics of an example aircraft are given in Table 1 and the thrust characteristics of the associated engine in Table 2. Calculations were carried out for a high-altitude hot

Table 1 Aircraft characteristics

Weight	5983 lb
Wing area	136 ft^2
$C_{L_{\max}}$	1.72
$C_{D_{\mathrm{roll}}}$	0.0880
$C_{L_{\mathrm{roll}}}$	0.60
Rolling friction coefficient	0.025
Braking friction coefficient	0.300

Table 2 Engine thrust characteristics

	V, knots	T, lb	Constant thrust, approx. lb
Twin engine takeoff	0.0	2007	1800
	68.7	1831	
	137.5	1699	
Single engine takeoff	0.0	1004	900
	68.7	916	
	137.5	850	
Single engine idle	0.0	45	20
	68.7	20	
	137.5	4	

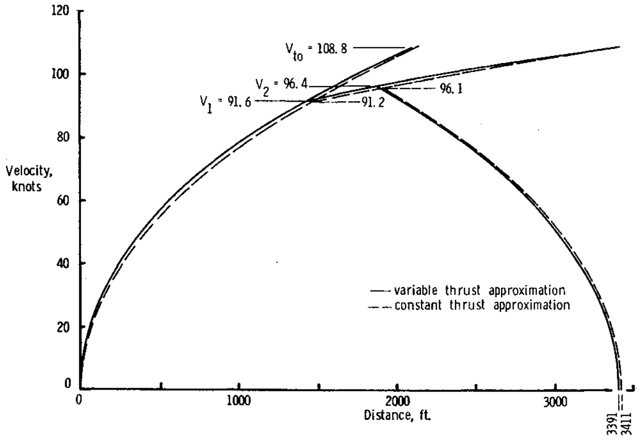

Fig. 2 Trainer takeoff performance: velocity as a function of distance.

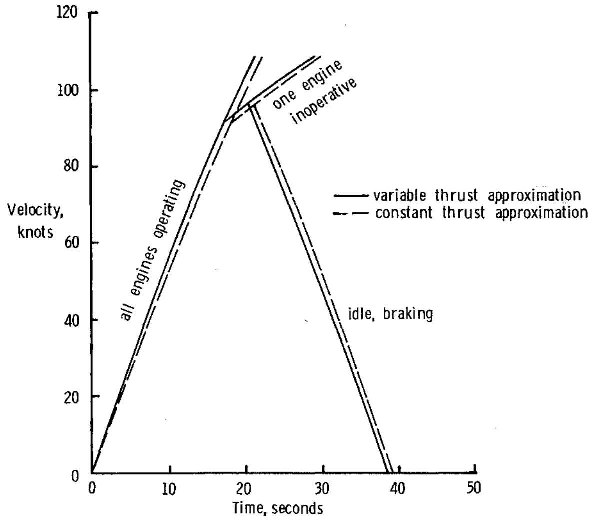

Fig. 3 Trainer takeoff performance: velocity as a function of time.

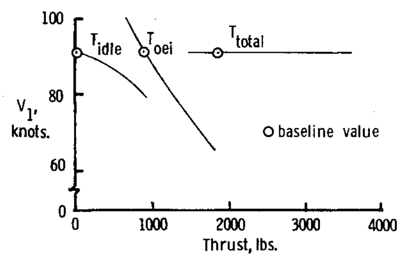

Fig. 4 Variation of critical engine failure speed with thrust levels.

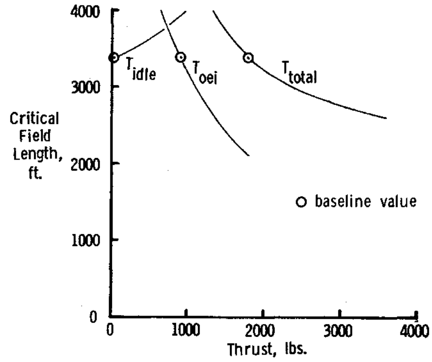

Fig. 5 Variation of critical field length with thrust levels.

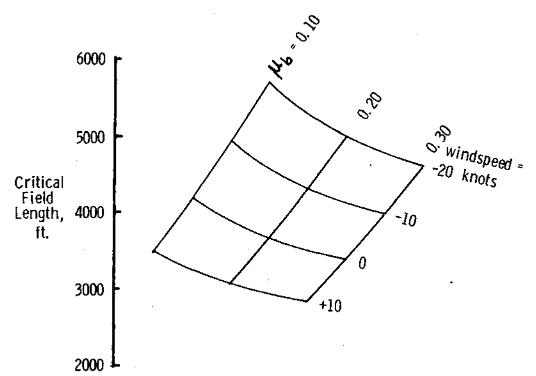

Fig. 6 Variation of critical field length with windspeed and braking friction.

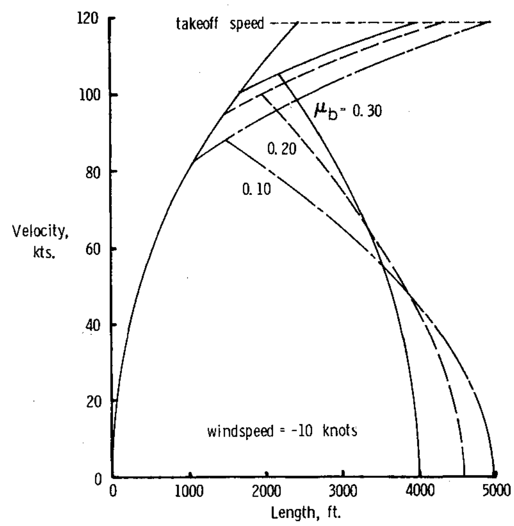

Fig. 7 Effect of braking friction on critical field length and critical engine failure speed.

day for which the density ratio was 0.7711. A time interval of 3 s was used between V_1 and V_2. Figure 2 shows the velocity-distance relationship for the given aircraft using both methods. Note that the critical field length given by the two methods differs by only 15 ft out of 3500. In addition, the critical engine failure speed V_1 as given by the two methods differs by only 0.4 knot, and V_2 by 0.3 knot. The twin-engine takeoff distances differ by only 40 ft. This agreement is excellent.

Figure 3 shows the velocity-time relationship. The largest error is approximately 1 s out of 40—a very acceptable agreement.

Figure 4 shows the variation of V_1 with changes in the thrust values used in the constant thrust approximation. The value of V_1 is not a function of the twin-engine takeoff thrust, as can be seen both from Fig. 4 and Eq. (23). The value of V_1 is a weak function of the idle thrust value used for curve 3. Increasing T_{idle} from 20 to 900 lb results in a decrease of V_1 from 91.2 to 79.3 knots, a change of 11.9 knots. The value of V_1 is a stronger function of the one-engine-inoperative thrust. Increasing T_{OEI} from 900 to 1800 lb decreases V_1 from 91.2 to 65.7 knots, a decrease of 25.5 knots. From these comparisons it can be seen that the constant thrust approximation of V_1 is not unduly sensitive to the choice of fixed thrust values.

Figure 5 shows the critical field lengths (CFL) associated with the values of V_1 given in Fig. 4. Note that while V_1 was not a function of the magnitude of the twin-engine thrust, the CFL certainly is. The higher the twin-engine thrust value, the shorter the CFL, as is to be expected. The same conclusion holds for the OEI thrust value. However, as the idle thrust

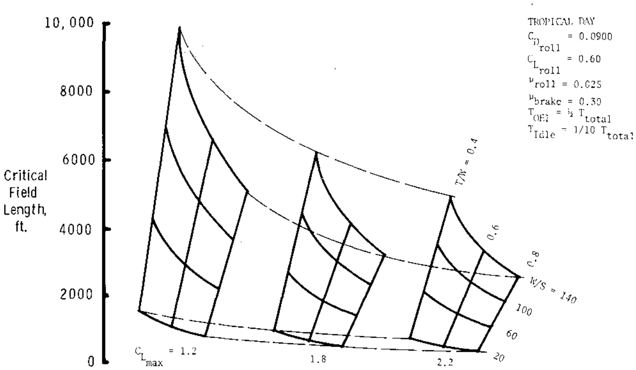

Fig. 8 Critical field length as a function of W/S, T/W, and $C_{L_{max}}$.

value increases so does the CFL, due to the increased braking distance. The sensitivity to error can be evaluated by increasing the individual thrust values by 20% one at a time. The resulting percent change in CFL from the basepoint value are: -8% for T_1, -10% for T_2, and $+0.05\%$ for T_3.

The effects of head winds and braking coefficients of friction on the CFL are shown in Fig. 6. The standard solution (zero head wind and $\mu_b = 0.3$) is a CFL of approximately 3200 ft. The CFL increments due to wind are essentially the same magnitude for both head and tail winds. The effects of changes in braking coefficient of friction are rather more nonlinear. A μ_b value of 0.10 (corresponding to wet runways) increases the CFL from 3200 to 4200 ft.

Figure 7 shows the corresponding velocity-distance plot for the case of a 10 knot tail wind. As the braking friction coefficient is reduced, V_1 is also reduced, due to the increased braking distance required to bring the aircraft to a halt.

The effects of wing loading, thrust loading, and $C_{L_{max}}$ are illustrated in Fig. 8 for an aircraft whose characteristics are given in the upper right-hand corner. Note that the loss of one engine is assumed to cut the available thrust in half, and the idle thrust is assumed to be one-tenth of the total thrust available. From this figure it can be seen that T/W is the dominant factor, as might be expected, closely followed by W/S. The influence of $C_{L_{max}}$ on the CFL increases with wing loading.

Discussion

The two methods discussed above were developed to allow a rapid evaluation of the critical field length capabilities of aircraft in the preliminary design process. The analysis is based upon the use of constant values of friction, lift, and drag coefficients in each segment of the analysis. In order to apply these methods to civil aircraft or to detailed analyses of "real world" performance problems, the effects of these assumptions should be evaluated. Other constraints, such as the influence of minimum control speed, have not been considered in this analysis. In addition, as pointed out by one of the referees, the Federal Air Regulations are more stringent in the types of accelerate-stop distances required.

Conclusions

Two analyses of the critical field length problem for jet aircraft have been presented. One includes the effects of thrust variations with relative speed and of head winds. The second method is based upon a constant thrust approximation and yet provides adequate accuracies for most preliminary design cases. In addition it can be programmed on a personal calculator. A program for this method is available through the Hewlett Packard User's Library as program 03631D.

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